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重尾分布和统计相依性在风险管理中的应用

Heavy-tailed Distribution And Statistical Dependency In Risk Management

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Heavy-tailed Distribution And Statistical Dependency In Risk Management

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重尾分布和统计相依性在风险管理中的应用

摘　要

重尾分布和统计相依性在风险管理中的应用是热点问题之一. 早在 1970 年以前, 统计研究发现诸多金融资产数据, 如棉花期货价格和股票收益等, 均展示出有别于正态分布的尖峰、重尾特征. 在具有正则变化分布的相依风险的条件下, 本文关注 Breiman 定理的理论推广及应用, 探讨带重尾索赔的多元破产概率问题, 并研究 Poisson shot-noise 过程尾部概率的渐近理论推导及应用.

首先, 关注 Breiman 定理的理论推广及应用. 在一个随机变量属于正则变化族而另一个随机变量满足 p 阶矩有限的条件下, Breiman 于 1965 年得到了关于两个非负独立随机变量乘积的 Breiman 定理. 随后诸多研究者在两随机变量间不独立或更弱的矩条件下对定理进行了推广. Breiman 定理的结论在风险管理中有重要应用, 例如其可用来刻画风险损失的随机折现. 本文在更一般的相依结构下, 借助 Karamata's 表示及勒贝格控制收敛定理, 推导出弱矩条件下的 Breiman 定理, 并利用新推导出的定理, 重温之前 Breiman 定理在相关领域中的应用.

其次, 讨论带重尾索赔的多元破产概率问题. 鉴于风险管理的实际需要, 破产概率的研究对保险公司来说具有重要意义. 对于一元破产问题, 过去几十年内已有诸多丰硕研究结果. 但对于多元问题, 在不同场景下, 基于各种破产集的定义有着多种版本的多元破产概率, 加上风险间有不可忽略的相依性, 关于多元破产概率的研究结果在过去二十年内较少. 本文以拥有 n 家子公司的保险公司为例, 在允许子公司的盈余资金进行不同比例自由转移的条件下, 定义新的多元破产集及相应的多元破产概率; 在各子公司面临的索赔服从重尾分布且具有一定

的统计相依性的条件下, 给出对应不同破产集的有限时间内的渐近破产概率, 并将其应用到再保险公司破产概率的估计中; 随后, 基于新定义的破产集及破产概率, 研究初始资金的分配策略并给出数值模拟实例.

最后, 研究具相依重尾冲击量的 Poisson shot-noise 过程的尾部概率的渐近性质. 近几十年来, shot-noise 被广泛应用于保险精算、 信用风险、 排队理论及系统可靠性等各领域, 它可用来描绘累积风险随时间动态发展的历程. 以往的研究关注冲击量与到达间隔相依的情形. 本文考虑重尾冲击量与冲击到达时刻具有一定统计相依的情形, 借助勒贝格控制收敛定理, 建立 Poisson shot-noise 过程尾概率的渐近理论, 并给出例子阐述主要定理的结论.

关键词： 正则变化; 勒贝格控制收敛定理; 多元破产概率; Copula; 上尾相依; 渐近独立

Heavy-tailed Distribution And Statistical Dependency In Risk Management

ABSTRACT

As one of the stylized decision problem, heavy-tailed distribution and statistical dependency in risk management has been paid significant attention. Statistical studies have found that data from financial assets, such as cotton futures price and stock returns, are presented spiked and heavy-tailed characteristics which are different from normal distribution. In the context of random risks with regular varying distributions and having some dependence structure, this dissertation focuses on the promotion and application of Breiman's theorem, discussing the multivariate ruin probability with heavy-tailed claims, and studying the tail behavior of the Poisson shot-noise processes.

Firstly, we focus on the promotion and application of Breiman's theorem. Under the conditions that one random variable is regularly varying and the other variable has finite p moment, Breiman built the theorem about the product of two nonnegative independent random variables in 1965. Subsequently, many researchers extended the result to dependent random variables or weaken moment condition. The results of Breiman's theorem have important applications in risk management, for example, it can be used to characterize the stochastic discount of loss. Under more general dependence structure and weaken moment condition, by virtue of Karamata's representation and Lebesgue dominated convergence theorem, we restart the Breiman's theorem. Some interesting applications are presented to illustrate the main result as well.

Secondly, we deal with the multivariate ruin probability with heavy-tailed claims. In view of the actual need for risk management, it's important for the insurance company to study the ruin probability. In the univariate case, the ruin probability has been intensively investigated in the past decades. But in the multivariate case, ruin may occur in various situations and hence there are several versions of the ruin probability

based on the corresponding ruin sets, fewer studies on the multivariate ruin probability have been conducted in past two decades. For an insurance company with n subcompany, in which capital transfers between subcompany are partially allowed, we denote some new multivariate ruin sets and the corresponding ruin probability. Under the framework of heavy-tailed claim amounts with some dependent structure, we derive the corresponding asymptotic finite-time ruin probability and apply them to estimate the ruin probability of reinsurance company. Then based on the new multivariate ruin sets and the corresponding ruin probability, we study the optimal allocation of the global initial reserve and present some numerical simulations results.

Finally, we study the tail behavior of the Poisson shot-noise processes with inter-dependent and heavy-tailed random shocks. In recent decades, the shot-noise process, which can be used to describe the dynamic development of cumulative risk over time, became very popular in various areas, such as insurance risk, credit risk, ad hoc network, queueing and reliability etc. Previous studies focused on shocks dependent of the inter-arrival times. In the presence of statistical dependence between shock and its arrival time, by virtue of Lebesgue dominated convergence theorem, we establish the asymptotic behavior of the tail probability of the Poisson shot-noise processes and some examples are presented as illustrations of the main results as well.

Key Words: Regular variation; Lebesgue dominated convergence theorem; Multivariate ruin probability; Copula; Upper tail dependence; Asymptotic independence

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